## Math 202 Final Exam

- The duration of the test is 2 hours
- No calculators are allowed

1. (15 points) Solve the following differential equation

$$
(x+2) \sin y d x+x \cos y d y=0 .
$$

2. (15 points) Consider the differential equation

$$
y^{\prime \prime}-5 y^{\prime}+6 y=e^{3 t}\left(5 t^{2}+t+1\right)
$$

Solve by first substituting $y=e^{3 t} u$ and then use the method of undetermined coefficients.
3. (15 points) Determine the singular points of the given differential equation and show that it is regular. Solve the differential equation using the power series solution using $\sum_{n=0}^{\infty} c_{n} x^{n+r}$ by finding first three terms of each solution.

$$
x y^{\prime \prime}+y^{\prime}+y=0
$$

4. (15 points) Use Laplace transform to solve the following initial value problem.

$$
y^{\prime \prime}+2 y^{\prime}+2 y=u(t-5)-u(t-20), \quad y(0)=0, y^{\prime}(0)=0
$$

5. (15 points) Use Laplace transform to find a solution of the following initial value problem

$$
y^{\prime \prime}+4 y=g(t), \quad y(0)=3, y^{\prime}(0)=-1
$$

where $g(t)$ is a continuous function of $t$.
6. (15 points) Use the method of undetermined coefficients to solve the following system of equations

$$
\begin{aligned}
& \frac{d x}{d t}=-4 x+y+z+3 \\
& \frac{d y}{d t}=x+5 y+-z+5 \\
& \frac{d z}{d t}=y-3 z-2
\end{aligned}
$$

7. ( 10 points) Let $S$ be the portion of the plane $x+y+z=1$ that lies in the first octant, and let $C$ be the boundary of $S$, traversed counterclockwise. Calculate $\int_{C} F$. $d r$ where $F=$ $-\frac{3}{2} y^{2} i-2 x y j+y z k$.
